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Technical Change and Welfare in an Open Economy with Distortions

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TECHNOLOGICAL CHANGE AND WELFARE IN AN OPEN ECONOMY WITH DISTORTIONS

RICHARD K. PERRIN AND LILYAN E. FULGINITI

Traditional measures of technological change, such as the rate of technical change, are based on producer-oriented prices. Here, we employ a general equilibrium analysis of an open economy to examine how the consumer welfare gain from a technological change, measured as Hicksian equivalent variation, is related to the rate of technical change, the biases of the technological change, and tax distortions in the economy. An analytical solution shows these relationships in a readily computable framework, and demonstrates that the rate of technical change will equal the rate of welfare change in only very unrealistic cases.

Key words: distortions, equivalent variation, general equilibrium, technological change.

Increases in productivity have clearly been the most important engine of real income gains during the past century, and it seems safe to assert that the productivity gains have been due primarily to technological change rather than to improved efficiencies in the use of existing technologies. Yet there has been remarkably little of the economic literature that has examined the characteristics of technological change and how these characteristics might affect the size and distribution of welfare benefits from technological innovation. The purpose of this article is to develop a general equilibrium model to explore the relationship between the characteristics and welfare implications of technological change and the effect of tax distortions on that relationship.

Two characteristics of technological change are crucial in determining its impacts: the rate of technological change and the biases it reflects toward individual inputs or outputs. The Solow residual, an estimate of the aggregate income gains from technological change, is one measure of the rate of technological change. Following Solow there was a substantial body of literature relating the residual to changes in the underlying aggregate production function and to index measures of the rate of technological change, with a loss of emphasis on the original welfare implications of the residual itself. Hicks introduced the notion of the input bias of technological change. He sketched out a model showing how, under constant prices, an input bias in technological change could change input shares, and this insight stimulated an extensive literature devoted to defining and measuring parameters representing such biases. The literature on rates and biases of technological change, however, stops short of an integrated evaluation of their price and income effects.

Literature on the welfare implication of particular technological changes has a long history. Griliches' study of the impact of hybrid corn technology was one of the first empirical efforts. The welfare measure he used was the change in social surplus (consumer and producer surplus) in the market for corn. Many partial equilibrium studies following Griliches examined the distribution of welfare benefits among related markets and how these benefits are affected by market imperfections (see Alston, Norton, and Pardey, for a review of this literature). Martin and Alston (1997), considering an economy with fixed prices, identified some circumstances in which traditional graphic producer surplus does not accurately reflect changes in profits resulting from technological change. Recently, Chavas and Cox developed a general equilibrium framework acknowledging in a general way the induced price effects of neutral technical change or price policies.

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Work on general equilibrium measures of welfare effects of technical change in the presence of policy distortions seems to be limited to Martin and Alston (1994), though their analysis was limited to the case of a small open economy in which there are no price impacts of the technological change. The closest in nature to the present task is the closed-economy general equilibrium model in Perrin and Fulginiti that examines the welfare effect of technical change in the presence of externalities.

A related vein of the literature has been directed at the issue of measuring productivity change, per se, in the presence of price distortions (discrepancies between prices and marginal productivities.) Distortions considered have been those from imperfect competition (see, for example, the studies in Cowing and Stevenson), from underutilization of capacity (see Berndt and Fuss and other papers in that special issue of the *Journal of Econometrics*), from economies of scale (Ohta), from pollution abatement regulations (Denison; Norsworthy, Harper, and Kunze; Crandall; Christiansen and Haveman; Pittman; Färe et al; and Conrad and Morrison), or from the existence of a common-property renewable resource (Capalbo). These producer-oriented studies focus on measuring productivity as a shift in the technology set, and thus carry the analysis away from the notion of how technological change affects the welfare of consumers.

The purpose of this article is to introduce an open-economy general equilibrium model of the consumer welfare impacts of technological change, and to show how these impacts are affected by the rate and biases of technological change and by price distortions.

General Equilibrium in an Open Economy

Consider an open economy with vectors of nontradable goods **y** and tradable goods **x**, for which a representative consumer expenditure function and an aggregate profit function can be represented by

$$(1)\quad E(\mathbf{p}+\mathbf{t}, \mathbf{w}+\mathbf{l}+\mathbf{s}, u) \equiv \mathrm{Min}_{\mathbf{y},\mathbf{x}^d}[(\mathbf{p}+\mathbf{t})\mathbf{y} + (\mathbf{w}+\mathbf{l}+\mathbf{s})\mathbf{x}^d \mid u(\mathbf{y}, \mathbf{x}^d) \geq u],$$

and

$$\Pi(\mathbf{p}, \mathbf{w}+\mathbf{l}, \tau) \equiv \mathrm{Max}_{\mathbf{y},\mathbf{x}^s} [\mathbf{p}\mathbf{y} + (\mathbf{w}+\mathbf{l})\mathbf{x}^s \mid (\mathbf{y}, \mathbf{x}^s, \tau) \in T]$$

where **y** is an $n \times 1$ vector of netput quantities of nontradables, **p** is an $n \times 1$ vector of producer prices for **y**, $\mathbf{x}^d$ is an $m \times 1$ vector of netput quantities of tradables demanded, $\mathbf{x}^s$ is an $m \times 1$ vector of netput quantities of tradables supplied, **w** is an $n \times 1$ vector of world prices for **x**, **t** is an $n \times 1$ vector of consumer taxes on nontradables, **l** is an $m \times 1$ vector of border taxes on tradables, (taxes on imports, subsidies of exports), **s** is an $m \times 1$ vector of consumer taxes on tradables, u is the representative consumer's utility function, τ is an index of technological change, T is the feasible technology set.

In this economy we assume that tax revenues are returned to consumers and that subsidy payments are taken from consumers (or, equivalently, that these amounts are costlessly allocated by the government in ways consistent with consumer preferences).

The general equilibrium conditions for a small open economy require that consumer expenditures must equal consumer income, and that nontradable quantities demanded and supplied must be equal. These equilibrium conditions may be represented by the following equations:

$$(2a)\quad E(\mathbf{p}+\mathbf{t}, \mathbf{w}+\mathbf{l}+\mathbf{s}, u) = \Pi(\mathbf{p}, \mathbf{w}+\mathbf{l}, \tau) + \mathbf{t}E_{\mathbf{p}} + (\mathbf{l}+\mathbf{s})\mathbf{E}_{\mathbf{w}} - \mathbf{l}\mathbf{\Pi}_{\mathbf{w}}$$

$$(2b)\quad \mathbf{E}_{\mathbf{p}} = \mathbf{\Pi}_{\mathbf{p}}.$$

Subscripts in equation (2) represent partial derivatives, so by Shephard's and Hotelling's lemmas these equations specify the equilibrium conditions described above. The above system of equations can be used to represent a closed economy by letting the tradable vector **x** be a scalar representing the numeraire good whose price does not change and can arbitrarily be set equal to unity. A representative initial equilibrium with these tax structures is depicted in figure 1.

We wish to consider an exogenous shock to this system in the form of a technological change from τ^0 to τ', as is illustrated in figure 2 for a simple two-good undistorted closed economy. The welfare effect of this shock to the system is the resulting equilibrium change in utility. Two approximations

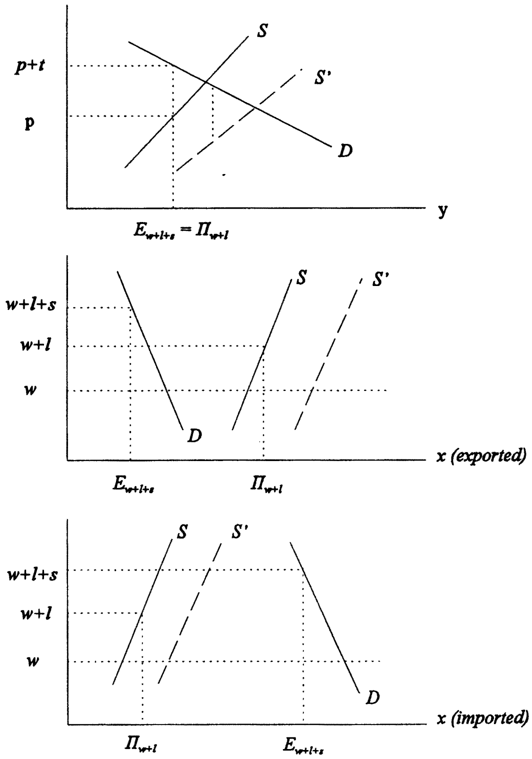

Figure 1. Tax implications at initial equilibrium (l, t, and s positive), with supply shifts

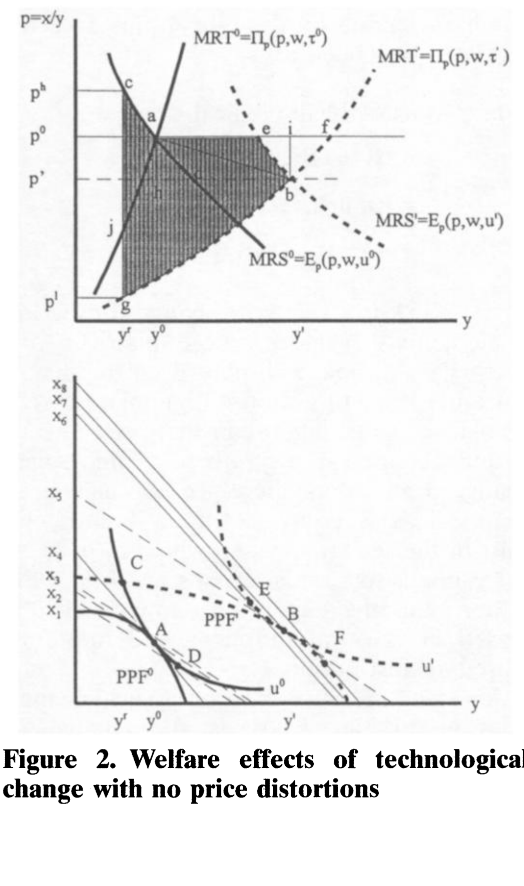

Figure 2. Welfare effects of technological change with no price distortions

of this welfare effect are the Hicks–Boiteux aggregate equivalent variation (EV) and compensating variation (CV) in income. In this article, we contrast these measures with two commonly used measures of technological change, the dual rate of technical change and total factor productivity.

Measures of Welfare Change

Hicks' EV measure of the welfare effect of a change from state A to state B (which was extended by Boiteux) is the minimum amount of money that if given to consumers in state A, would permit the consumer to achieve the utility level of state B. CV is the maximum amount of money that could be taken away at state B and still permit the consumer to achieve the utility level of state A. In terms of the economy described above, if the technological change moved the economy from an initial equilibrium ($\mathbf{p}^0$, $\mathbf{w}$, u^0) to ($\mathbf{p}'$, $\mathbf{w}$, u') while fixing taxes at $\mathbf{t}$, $\mathbf{l}$, and $\mathbf{s}$, then EV is defined as

(3) Equivalent Variation (EV)

$$\equiv E(\mathbf{p}^0 + \mathbf{t}, \mathbf{w} + \mathbf{l} + \mathbf{s}, u') - E(\mathbf{p}^0 + \mathbf{t}, \mathbf{w} + \mathbf{l} + \mathbf{s}, u^0)$$

and CV is defined as

(4) Compensating Variation (CV)

$$\equiv E(\mathbf{p}' + \mathbf{t}, \mathbf{w} + \mathbf{l} + \mathbf{s}, u') - E(\mathbf{p}' + \mathbf{t}, \mathbf{w} + \mathbf{l} + \mathbf{s}, u^0).$$

Figure 2 represents a two-good closed economy in which these concepts can be contrasted graphically. The lower panel illustrates the numeraire good $\mathbf{x}$ on the vertical axis, the other good $\mathbf{y}$ on the horizontal axis, an initial technology and initial welfare level represented by PPF^0 and u^0, and a subsequent technology and welfare level represented by PPF$'$ and u'. In the upper panel, the MRS curves (Hicksian demand schedules) are negative slopes of the respective indifference curves in the lower panel, and the MRT curves (supply schedules) are negative slopes of the production possibility curves. The initial equilibrium at point A in the lower panel corresponds to point a in the upper panel. EV corresponds to $x_6 - x_4$ (which is shown in the Appendix to equal the shaded area *caebg* in the upper panel), while CV corresponds to the distance $x_5 - x_1$ and area *cadbg*. The reference point C is a combination of goods on the frontier of the new technology providing exactly the level of welfare as the initial equilibrium.

This graphic representation of the welfare effect of technological change in a general equilibrium framework provides an interesting contrast to the traditional partial equilibrium representation, which is the entire area between the two supply curves and under the initial demand curve.[1] Measurement of the traditional partial equilibrium welfare effect requires knowledge of both the new and the old supply functions throughout their domain, and the nature of the supply shift near the origin (pivotal, convergent, or divergent) has significant impact on welfare measurement,[2] but is not readily measurable. In contrast, the general equilibrium measure here requires only local approximations of demand and supply curves in the vicinity of the two equilibria.

Measures of Technological Change

The literature on technological change has tended quite naturally to be technology- or production-oriented rather than welfare-oriented, focusing on scalar measures of the *rate* of change that characterize such changes as the one from PPF^0 to PPF$'$ in figure 2. In general, no unique scalar value measures the enlargement of the production set. One commonly used measure of the rate of technical change is the relative change in maximum profit for a given set of prices. To provide a money-metric measure that we can compare to EV and CV, we consider the change in level of profit and refer to it as technical change (TC) as well as the rate of technical change (RTC) represented also with the symbol δ:

(5) Technical Change (TC)
$$\equiv \Pi_\tau(\mathbf{p}^0, \mathbf{w}+\mathbf{l}, \tau^0)d\tau$$
$$= \Pi(\mathbf{p}^0, \mathbf{w}+\mathbf{l}, \tau')$$
$$-\Pi(\mathbf{p}^0, \mathbf{w}+\mathbf{l}, \tau^0)$$
Rate of Technical Change (RTC)
$$\equiv \delta \equiv \frac{\Pi_\tau(\mathbf{p}^0, \mathbf{w}+\mathbf{l}, \tau)d\tau}{\Pi(\mathbf{p}^0, \mathbf{w}+\mathbf{l}, \tau^0)}.$$

In terms of figure 2, technical change is equivalent to $(x_8 - x_4)$ as evaluated at initial prices, or area *gcaf* (see the Appendix.)

The nature of technological change can be characterized by bias as well as rate, as originally suggested by Hicks. Here, we use the Binswanger definition and define netput bias as the percentage change in the share of netput in profit ($k_{yi} = p_i y_i/\Pi$, $k_i = p_i x_i/\Pi$) due to the technological change under constant prices, $\beta_{\mathbf{y}i} = d\ln k_{yi}/d\tau$, $\beta_{\mathbf{x}i} = d\ln k_{xi}/d\tau$. It is easily shown that share-weighted biases so defined must sum to zero, and that they may also be expressed as $\beta_i = \Pi_{i\tau}/y_i - \delta$; that is, the difference between the rate of change in optimal netput y_i or x_i and the rate of change in profit. Thus, the technological change can be characterized by the rate δ plus a vector of biases $\mathbf{B}$ defined as

(6) Bias of Technological Change
$$\equiv \mathbf{B} \equiv (\mathbf{B}_\mathbf{y}, \mathbf{B}_\mathbf{x})^T$$
$$\equiv (d\ln \mathbf{k}_\mathbf{y}, d\ln \mathbf{k}_\mathbf{x})^T$$
$$= (\hat{\mathbf{y}}^{-1}\mathbf{\Pi}_{\mathbf{p}\tau} - \iota\delta, \hat{\mathbf{x}}^{-1}\mathbf{\Pi}_{\mathbf{w}\tau} - \iota\delta)^T$$

where $d\ln\mathbf{k}$ is a vector of changes in the log of elements k_i, $\hat{\mathbf{y}}$ and $\hat{\mathbf{x}}$ indicate matrices with vectors $\mathbf{y}$ and bold $\mathbf{x}$ displayed on the diagonal, and ι is a unit vector. With unbiased technological change under constant prices, every netput changes at the rate of technological change δ and, thus, there are no changes in shares. This is equivalent to a homothetic shift in the technology set, which in figure 2 corresponds to a radial expansion of the PPF, rather than the expansion shown which is biased in favor of output y and therefore biased against output x.

A second measure of technological change relevant to this study is the change in total factor productivity as measured with a Törnquist–Theil index. We define the level of

[1] Martin and Alston (1997), for example, provide comparable graphs for fixed prices.

[2] Alston, Norton, and Pardey and Martin and Alston (1994) present lengthy discussions of the implications of parallel versus pivotal supply shifts on total research benefits. They conclude that total research benefits for a proportional shift is approximately half of that for an equivalent parallel shift and that producers can never lose from a parallel supply shift when demand is inelastic while they necessarily lose from a proportional supply shift.

this change and its rate as

(7) Factor Productivity Change (FPC)

$$\equiv \Pi(\mathbf{p}, \mathbf{w}+\mathbf{l}, \tau^0) \times [\tfrac{1}{2}(\mathbf{k}^0 + \mathbf{k}')(d\ln\mathbf{y}, d\ln\mathbf{x})^T]$$

Rate of Factor Productivity Change (RFPC)

$$\equiv \tfrac{1}{2}(\mathbf{k}^0 + \mathbf{k}')(d\ln\mathbf{y}, d\ln\mathbf{x})^T$$

where $\mathbf{k}^0 = (\mathbf{k}_\mathbf{y}, \mathbf{k}_\mathbf{x})$ is a vector of initial netput shares in profit, $\mathbf{k}'$ is a vector of subsequent netput values divided by initial profit.[3] In the two-good case illustrated in figure 2, FPC corresponds to area *gcab*, close to the average of CV and EV. Diewert has previously confirmed that the Törnquist–Theil index is an approximation to welfare change that is exact for certain expenditure functions.

To this point, we distinguished four possible measures of the benefits of technological change. The two-good illustrations of figure 2 provide an intuitive appreciation of their differences in a nondistorted economy. The differences between the measures appear here in the form of triangles *abd*, *aeb* and *efb*. The differences between CV, EV, TC, and FPC will disappear if there are no induced price changes, which would be the case if technical change has no bias and preferences are homothetic. When price distortions are introduced, the differences between the measures will be more complex. To explore these relationships more fully requires establishing the induced price effects of the technological change, both with and without price distortions. For this purpose, we resort to the general equilibrium conditions of equation (2) to examine the comparative static implications of technological change.

Comparative Statics of Technological Change

We demonstrated the general equilibrium welfare effects of technological change graphically in the two-good case. To quantify these effects algebraically, we examine the comparative statics of the equilibrium system described in equation (2). The total differentials of these equilibrium conditions can be represented as

(8a)
$$[\mathbf{E}_u - \mathbf{t}\mathbf{E}_{\mathbf{p}u} - (\mathbf{l}+\mathbf{s})\mathbf{E}_{\mathbf{w}u}]\,du + [-\mathbf{t}\mathbf{E}_{\mathbf{pp}} - (\mathbf{l}+\mathbf{s})\mathbf{E}_{\mathbf{wp}} + \mathbf{l}\mathbf{\Pi}_{\mathbf{wp}}]\,d\mathbf{p} = [\mathbf{\Pi}_\tau - \mathbf{l}\mathbf{\Pi}_{\mathbf{w}\tau}]\,d\tau$$

(8b)
$$[\mathbf{E}_{\mathbf{p}u}]du + [\mathbf{E}_{\mathbf{pp}} - \mathbf{\Pi}_{\mathbf{pp}}]\,d\mathbf{p} = \mathbf{\Pi}_{\mathbf{p}\tau}\,d\tau.$$

Substituting earlier definitions, these may be rewritten in elasticity form as

(9a)
$$[1 - \mathbf{T}\boldsymbol{\varepsilon}]d\ln u - \boldsymbol{\xi} d\ln\mathbf{p} = [(1 - \mathbf{T}\boldsymbol{\iota})\delta - \boldsymbol{\theta}\mathbf{B}_\mathbf{x}]\,d\tau$$

(9b)
$$\boldsymbol{\varepsilon}_y\, d\ln u + [\mathbf{H}_{\mathbf{pp}} - \mathbf{\Sigma}_{\mathbf{pp}}]\,d\ln\mathbf{p} = [\mathbf{B}_\mathbf{y} + \boldsymbol{\iota}_\mathbf{y}\delta]\,d\tau$$

where $\mathbf{T} \equiv (\mathbf{T}_\mathbf{y}, \mathbf{T}_\mathbf{x}) \equiv$ vector of consumer tax payments as a fraction of expenditures $= (\|(1/E)t_iE_{\mathbf{p}i}\|, \|(1/E)(l_i + s_i)E_{\mathbf{w}i}\|) = ((1/E)\mathbf{t}\widehat{\mathbf{E}}_\mathbf{p}, (1/E)(\mathbf{l} + \mathbf{s})\widehat{\mathbf{E}}_\mathbf{w})$, $\boldsymbol{\theta} \equiv$ vector of producer tax payments as a fraction of expenditures $= \|(1/E)l_i\Pi_{\mathbf{w}i}\| = (1/E)l\mathbf{\Pi}_\mathbf{w}$, $\boldsymbol{\varepsilon} \equiv (\boldsymbol{\varepsilon}_\mathbf{y}, \boldsymbol{\varepsilon}_\mathbf{x}) \equiv$ vector of income elasticities of demand $= \|d\ln E_i/d\ln u\| = (u\widehat{\mathbf{E}}_\mathbf{p}^{-1}\mathbf{E}_{\mathbf{p}u}, u\widehat{\mathbf{E}}_\mathbf{w}^{-1}\mathbf{E}_{\mathbf{w}u})$, $\mathbf{\Sigma}_\mathbf{ij} \equiv$ submatrix of supply elasticities $= \widehat{\mathbf{\Pi}}_\mathbf{i}^{-1}\mathbf{\Pi}_\mathbf{ij}\hat{\mathbf{j}}$, $\mathbf{i}, \mathbf{j} = \mathbf{p}, \mathbf{w}$, $\boldsymbol{H}_{ij} \equiv$ submatrix of demand elasticities $= \widehat{\mathbf{E}}_\mathbf{i}^{-1}\mathbf{E}_\mathbf{ij}\hat{\mathbf{j}}$, $\mathbf{i}, \mathbf{j} = \mathbf{p}, \mathbf{w}$, and $\boldsymbol{\xi} \equiv$ tax-share weighted price elasticities $\mathbf{T}_\mathbf{y}\boldsymbol{H}_{\mathbf{pp}} + \mathbf{T}_\mathbf{x}\boldsymbol{H}_{\mathbf{wp}} - \boldsymbol{\theta}\mathbf{\Sigma}_{\mathbf{wp}}$, where ^ represent matrices with the indicated vector displayed on the diagonal, as before. Solving equation (9a) for $d\ln u$ and substituting into equation (9b), these equations may be solved to express the price and the quantity effects of technological change in terms of its rate, δ, and biases $\mathbf{B}_\mathbf{y}$ and $\mathbf{B}_\mathbf{x}$. The technology-induced changes in the prices of tradables is

(10)
$$d\ln\mathbf{p}/d\tau = \{\mathbf{\Gamma}^{-1}\mathbf{L}\}\delta - \{\mathbf{\Gamma}^{-1}\}\mathbf{B}_\mathbf{y} - \left\{\frac{1}{1 - \mathbf{T}\boldsymbol{\varepsilon}}\mathbf{\Gamma}^{-1}\boldsymbol{\varepsilon}_\mathbf{y}\boldsymbol{\theta}\right\}\mathbf{B}_\mathbf{x}$$

where $\mathbf{L} \equiv (1 - \mathbf{T}\boldsymbol{\iota})/(1 - \mathbf{T}\boldsymbol{\varepsilon})\boldsymbol{\varepsilon}_\mathbf{y} - \boldsymbol{\iota}$, and $\mathbf{\Gamma} \equiv \mathbf{\Sigma}_{\mathbf{pp}} - \boldsymbol{H}_{\mathbf{pp}} - 1/(1 - \mathbf{T}\boldsymbol{\varepsilon})\boldsymbol{\varepsilon}_\mathbf{y}\boldsymbol{\xi}$. The changes in equilibrium quantities of nontradable goods

[3] This alteration of the Törnquist–Theil productivity index simplifies the conversion of rates of change to levels of change.

demanded and supplied is

$$(11)\quad d\ln\mathbf{y}/d\tau = \left\{\frac{1-\mathbf{T}\boldsymbol{\iota}}{1-\mathbf{T}\boldsymbol{\varepsilon}}\boldsymbol{\varepsilon}_{\mathbf{y}}\left(\boldsymbol{H}_{\mathbf{pp}}+\frac{1}{1-\mathbf{T}\boldsymbol{\varepsilon}}\boldsymbol{\varepsilon}_{\mathbf{y}}\boldsymbol{\xi}\right)\times\boldsymbol{\Gamma}^{-1}\mathbf{L}\right\}\delta - \left\{\boldsymbol{H}_{\mathbf{pp}}+\frac{1}{1-\mathbf{T}\boldsymbol{\varepsilon}}\boldsymbol{\varepsilon}_{\mathbf{y}}\right\}\boldsymbol{\Gamma}^{-1}\mathbf{B}_{\mathbf{y}} - \frac{1}{1-\mathbf{T}\boldsymbol{\varepsilon}}\left\{\left(\boldsymbol{H}_{\mathbf{pp}}+\frac{1}{1-\mathbf{T}\boldsymbol{\varepsilon}}\boldsymbol{\varepsilon}_{\mathbf{y}}\right)\times\boldsymbol{\Gamma}^{-1}+\mathbf{I}\right\}\boldsymbol{\varepsilon}_{\mathbf{y}}\boldsymbol{\theta}\mathbf{B}_{\mathbf{x}}$$

while changes in equilibrium quantities of tradables demanded and supplied are

$$(12)\quad d\ln\mathbf{x}^{d}/d\tau = \left\{\frac{1-\mathbf{T}\boldsymbol{\iota}}{1-\mathbf{T}\boldsymbol{\varepsilon}}\boldsymbol{\varepsilon}_{\mathbf{x}} + \left(\boldsymbol{H}_{\mathbf{wp}}+\frac{1}{1-\mathbf{T}\boldsymbol{\varepsilon}}\boldsymbol{\varepsilon}_{\mathbf{x}}\boldsymbol{\xi}\right)\boldsymbol{\Gamma}^{-1}\mathbf{L}\right\}\delta - \left\{\boldsymbol{H}_{\mathbf{wp}}+\frac{1}{1-\mathbf{T}\boldsymbol{\varepsilon}}\boldsymbol{\varepsilon}_{\mathbf{x}}\boldsymbol{\xi}\right\}\boldsymbol{\Gamma}^{-1}\mathbf{B}_{\mathbf{y}} - \frac{1}{1-\mathbf{T}\boldsymbol{\varepsilon}}\left\{\left(\boldsymbol{H}_{\mathbf{wp}}+\frac{1}{1-\mathbf{T}\boldsymbol{\varepsilon}}\boldsymbol{\varepsilon}_{\mathbf{x}}\boldsymbol{\xi}\right)\times\boldsymbol{\Gamma}^{-1}\boldsymbol{\varepsilon}_{\mathbf{y}}+\boldsymbol{\varepsilon}_{\mathbf{x}}\right\}\boldsymbol{\theta}\mathbf{B}_{\mathbf{x}}$$

$$d\ln\mathbf{x}^{s}/d\tau = \left\{\boldsymbol{\iota}+\boldsymbol{\Sigma}_{\mathbf{wp}}\boldsymbol{\Gamma}^{-1}\mathbf{L}\right\}\delta - \left\{\boldsymbol{\Sigma}_{\mathbf{wp}}\boldsymbol{\Gamma}^{-1}\right\}\mathbf{B}_{\mathbf{y}} + \left\{\mathbf{I}-\frac{1}{1-\mathbf{T}\boldsymbol{\varepsilon}}\boldsymbol{\Sigma}_{\mathbf{wp}}\boldsymbol{\Gamma}^{-1}\boldsymbol{\varepsilon}_{\mathbf{y}}\boldsymbol{\theta}\right\}\mathbf{B}_{\mathbf{x}}.$$

From equation (10), it is clear that there are two conditions that together are sufficient for the technological change to have no induced price effects on the nontradables: if all income elasticities of demand are unitary (all $\varepsilon_i = 1$, in which case $\mathbf{L} = \mathbf{0}$) *and* if technological change is unbiased (all $\boldsymbol{\beta}_{\mathbf{i}} = \mathbf{0}$). This algebraic result can be seen for the two-good case of figure 2, where if the PPF shifts radially (no biases) and the successive indifference curves are radial expansions (preferences are homothetic) there will clearly be no price effects of the technical change.

We can now return to the issue of whether price distortions have an important impact on the way we measure technological change. Using a Taylor expansion of $E(\mathbf{p}^0+\mathbf{t}, \mathbf{w}+\mathbf{l}+\mathbf{s}, u')$ about the equilibrium point $E(\mathbf{p}'+\mathbf{t}, \mathbf{w}+\mathbf{l}+\mathbf{s}, u')$, we obtain a second-order approximation of the EV associated with technological change:

$$\begin{aligned}(13)\quad \mathrm{EV} &= E(\mathbf{p}^0+\mathbf{t}, \mathbf{w}+\mathbf{l}+\mathbf{s}, u') - E(\mathbf{p}^0+\mathbf{t}, \mathbf{w}+\mathbf{l}+\mathbf{s}, u^0)\\ &\cong E(\mathbf{p}'+\mathbf{t}, \mathbf{w}+\mathbf{l}+\mathbf{s}, u') + \mathbf{E}_{\mathbf{p}}d\mathbf{p} + \tfrac{1}{2}d\mathbf{p}^T \times \mathbf{E}_{\mathbf{pp}}(\mathbf{p}', \mathbf{w}, u')d\mathbf{p} - E(\mathbf{p}^0+\mathbf{t}, \mathbf{w}+\mathbf{l}+\mathbf{s}, u^0)\\ &= (\mathbf{p}'+\mathbf{t})\mathbf{y}' + (\mathbf{w}+\mathbf{l}+\mathbf{s})\mathbf{x}' + \mathbf{y}'(\mathbf{p}^0-\mathbf{p}') + \tfrac{1}{2}d\mathbf{p}^T \times \mathbf{E}_{\mathbf{pp}}(\mathbf{p}', \mathbf{w}, u')d\mathbf{p} - (\mathbf{p}^0+\mathbf{t})\mathbf{y}^0 - (\mathbf{w}+\mathbf{l}+\mathbf{s})\mathbf{x}^0\\ &= (\mathbf{p}^0+\mathbf{t})d\mathbf{y} + (\mathbf{w}+\mathbf{l}+\mathbf{s})d\mathbf{x} + \tfrac{1}{2}d\mathbf{p}^T\mathbf{E}_{\mathbf{pp}}(\mathbf{p}', \mathbf{w}, u')d\mathbf{p}.\end{aligned}$$

Substituting changes in quantities and prices from equations (10)–(12), EV, expressed as a fraction of initial expenditures $E^0 = E(\mathbf{p}^0 + \mathbf{t}, \mathbf{w}+\mathbf{l}+\mathbf{s}, u^0)$, is

$$\begin{aligned}(14)\quad \mathrm{EV}/E^0 &= \delta + \frac{1}{1-\mathbf{T}\boldsymbol{\varepsilon}} \times \left\{\mathbf{T}(\boldsymbol{\varepsilon}-\boldsymbol{\iota}) + \boldsymbol{\xi}\boldsymbol{\Gamma}^{-1}\mathbf{L}\right\}\delta - \frac{1}{1-\mathbf{T}\boldsymbol{\varepsilon}}\{\boldsymbol{\xi}\boldsymbol{\Gamma}^{-1}\}\mathbf{B}_{\mathbf{y}} - \frac{1}{1-\mathbf{T}\boldsymbol{\varepsilon}} \times \left\{1+\frac{1}{1-\mathbf{T}\boldsymbol{\varepsilon}}\boldsymbol{\xi}\boldsymbol{\Gamma}^{-1}\boldsymbol{\varepsilon}_{\mathbf{y}}\right\}\boldsymbol{\theta}\mathbf{B}_{\mathbf{x}}\\ &\quad + \frac{1}{2}\left[\mathbf{L}\delta - \mathbf{B}_{\mathbf{y}} - \frac{1}{1-\mathbf{T}\boldsymbol{\varepsilon}}\boldsymbol{\varepsilon}_{\mathbf{y}}\boldsymbol{\theta}\boldsymbol{\beta}_{\mathbf{x}}\right]^T \times \mathbf{Z}\left[\mathbf{L}\delta - \mathbf{B}_{\mathbf{y}} - \frac{1}{1-\mathbf{T}\boldsymbol{\varepsilon}}\boldsymbol{\varepsilon}_{\mathbf{y}}\boldsymbol{\theta}\boldsymbol{\beta}_{\mathbf{x}}\right]\end{aligned}$$

where $\mathbf{Z} \equiv (\boldsymbol{\Gamma}^{-1})^T\widehat{S}_{\mathbf{y}}\boldsymbol{H}_{\mathbf{pp}}\boldsymbol{\Gamma}^{-1}$, and $\widehat{S}_{\mathbf{y}}$ is the diagonal matrix with expenditure shares on $\mathbf{y}$ on the diagonal.

Equation (14) is the primary analytical result of this article, an explicit solution of the consumers' welfare gains[4] from technical change, as a function of the rate and

[4] A similar expression can be derived for the CV measure of welfare benefit.

bias characteristics of that technical change, conditional on three kinds of tax–subsidy distortions. The special case of equation (14) for an undistorted economy is

$$\text{(14a)}\quad \mathrm{EV}/E^0 = \delta + (1/2)(\boldsymbol{\varepsilon} - \boldsymbol{\iota})^T\mathbf{Z}(\boldsymbol{\varepsilon} - \boldsymbol{\iota})\delta^2 + (1/2)\mathbf{B}_y^T\mathbf{Z}\mathbf{B}_y - (\boldsymbol{\varepsilon} - \boldsymbol{\iota})^T\mathbf{Z}\mathbf{B}_y\delta$$

while the special case of equation (14) for no technology-induced price changes may be derived as

$$\text{(14b)}\quad \mathrm{EV}/E^0 = \delta + \frac{1}{1-\mathbf{T}\boldsymbol{\varepsilon}}\mathbf{T}(\boldsymbol{\varepsilon} - \boldsymbol{\iota})\delta - \frac{1}{1-\mathbf{T}\boldsymbol{\varepsilon}}\boldsymbol{\theta}\mathbf{B}_x.$$

The last two equations establish that either induced price effects or distortions independently can cause a discrepancy between the rate of technical change and its welfare impact. The task now is to consider the conditions under which there is no discrepancy between δ and EV.

First, it is evident from equation (14) that the rate of technical change, δ, will equal the rate of welfare gain if (*a*) technological change is unbiased ($\mathbf{B} = \mathbf{0}$) and (*b*) consumer preferences are homothetic ($\boldsymbol{\varepsilon} = \boldsymbol{\iota}$). We refer to this as the homothetic case, and the conclusion is true regardless of distortions. This conclusion demonstrates algebraically what was intuitively clear from the previous graphic analysis of the two-good undistorted case: when the technical change is a radial expansion of the PPF, and indifference curves are radial expansions of one another, there are no induced price effects and the rate of technical change is an unbiased measure of the welfare effect of technological change.

Second, it is evident from equation (14b) that when there are no induced price effects, the rate of technical change, δ, will equal the rate of welfare gain in an undistorted economy (i.e., when $\mathbf{T} = \mathbf{0}$ and $\boldsymbol{\theta} = \mathbf{0}$)[5]. The necessary condition for no induced price effects is, from equation (10), $\boldsymbol{\Gamma}^{-1}\mathbf{L}\delta = \boldsymbol{\Gamma}^{-1}[\mathbf{B}_y + \boldsymbol{\varepsilon}_y\boldsymbol{\theta}\mathbf{B}_x/(1 - \mathbf{T}\boldsymbol{\varepsilon})]$; that is, any price changes induced by the biases in technical change must be exactly offset by the nonhomotheticity of preferences. One special sufficient condition for this to hold is the homotheticity condition described above. In general, it seems highly unlikely that the conditions for no induced price effects will be met.

However, one other special sufficient condition for no price effects is of interest, the Martin–Alston (1994, 1997) case of a small country with prices exogenous because all commodities are tradables. (Either supply or demand elasticities must in this case become infinite, insuring that equation (10) equals zero.) From equation (14b), we have seen that the rates of welfare change and technical change will be equal for this open economy only if there are no distortions. However, if the only distortion is a relatively small domestic tax $t_i(\boldsymbol{\theta} = \mathbf{0})$ that has been levied on a luxury good, such that $T_i^{-1} > \varepsilon_i > 1$, then the coefficient of δ exceeds unity, and the tax itself causes the welfare effect to be greater than the rate of technical change. Similarly, a single border tax l_i (a subsidy if x_i is exported, a tax if it is imported) will also enhance welfare effect of technical change if it is levied on a commodity toward which the technical change is biased. A border tax will enhance welfare effects of technical change if it has the same sign as the bias, and reduce welfare effects otherwise.

In their examination of the exogenous-price case, Martin and Alston (1994) concluded that the rate of technical change may be a misleading measure of the welfare benefits of technical change. The analysis here, which holds for the more general case that includes nontradables, confirms that conclusion and in addition shows *when* the rate of technical change will be a misleading measure, namely, when the technological change is biased or when preferences are nonhomothetic. Their numerical simulations employed a negative border tax on cocoa and a positive one on rice, both being goods toward which technical change was biased, and rice with a low income elasticity of demand. They found in those specific examples that welfare gains were considerably smaller than the rate of technical change, consistent with the results of the general analysis presented here.

One additional special case of equation (14) warrants some consideration, and that is the case of a two-good, closed, and distorted economy (represented in figure 2 without the distortion.) In this case, the good x, whose price does not change, is interpreted as the numeraire good, while y is the good whose price adjusts. Equation (14) then reduces to

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[5] This is also inherent in (14a) combined with the special case of no price effects.

the scalar quadratic equation,

$$(15)\quad EV/E^0 = \left\{ \frac{1-T_y}{1-T_y\varepsilon_y} + \frac{T_y\eta_{yy}(\varepsilon_y-1)}{\sigma_{yy}-\eta_{yy}-T_y\varepsilon_y\sigma_{yy}} \right\}\delta + \frac{-T_y\eta_{yy}(\varepsilon_y)}{\sigma_{yy}-\eta_{yy}-T_y\varepsilon_y\sigma_{yy}}B_y + \frac{s_y\eta_{yy}}{2(\sigma_{yy}-\eta_{yy}-T_y\varepsilon_y\sigma_{yy})^2} \times [(\varepsilon_y-1)\delta+B_y]^2.$$

The rate of welfare change differs from the rate of technical change due both to tax distortions and to bias in the technical change. For a positive tax on y, the coefficient of the second term will be positive (unless $T_y\varepsilon_y\sigma_{yy}$ is sufficiently large), so the rate of welfare change will be augmented if the technical change is biased toward the good that is taxed. It is evident that the final term must be negative (because η_{yy} must be negative), reflecting negative second-order effects.

We have considered the conditions under which δ is an unbiased measure of the welfare effect of technical change. We found that this happens in three very unlikely cases:

(1) homothetic preferences and neutral technical change,
(2) when the economy has no policy distortions and the technical change causes no induced price effects, and
(3) when a combination of parameter values eliminates all but the first term on the right-hand side of equation (14).

We have also shown that:

(1) when technical change induces price changes in the economy, the rate of technical change will likely differ from the welfare measurement of technical change, and
(2) in the presence of policy distortions, the rate of technical change will likely differ from its welfare impact, regardless of the effect on prices of such change.

In two special cases, we are able to determine the effect of distortions on the welfare benefit from technology. In the small country case, a domestic tax on any one commodity will increase the technology benefits under plausible conditions, and a border tax on a single commodity will increase the benefits if it is levied on a commodity toward which the technical change is biased. In the two-good economy, a tax will increase benefits if the technical change is biased toward the taxed good.

The general case described by equation (14), however, is too complex to permit us to sign the effects of a particular tax on the benefits from technological change, and does not permit generalizations concerning the rate of welfare change relative to the rate of technical change. A general welfare principle is that any distortion decreases welfare relative to a Pareto optimum. However, when a distortion is combined with technical change, welfare does not necessarily decrease from that Pareto optimum. It is as if the technical change effect on welfare could enhance or diminish the waste due to the distortionary effect.

Conclusions

This article uses a general equilibrium model to derive an algebraic solution for the rate of welfare gain from a technical change, in terms of the rate and biases of the change and the size of any tax wedges on tradables or nontradables. The algebraic structure of the solution provides a simple method of computing the price and welfare effects of technical change in general equilibrium. The main analytical conclusion derived from the analysis is that the rate of technological change, as usually measured from the production perspective, will hardly ever be an unbiased measure of the welfare benefits of technical change. The discrepancy arises because prices of domestic goods may be altered (due either to biased technical change or to nonhomothetic preferences), or because of the effects of price distortions in the form of domestic and/or border taxes.

This article contributes to the existing literature in three ways: (a) it emphasizes the importance of measuring the impact of technical change on consumers rather than producers and it finds that only under very restrictive conditions the producers' rate of technical change will reflect the consumers' welfare change; (b) it extends the literature on welfare effects of technical change to an open economy general equilibrium approach that identifies induced price effects due to an innovation; and (c) it analyzes the effect of multiple price distortions on the welfare implications of technological change. These contributions are clearly and concisely presented in a single comparative statics equation (14), which for the first time expresses the welfare effects of technical change in terms of traditional producers' measures, the rate of technical change and its vector of biases.

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Appendix

This appendix establishes the equivalence of the various welfare measures to areas in case of a two-good, undistorted closed economy ($w = 1$) as depicted in figure 2. First, consider the equivalence of EV ($x_6 - y_4$) and area *caebg*:

(A1) $\mathrm{EV} = E(p^0, w, u') - E(p^0, w, u^0)$

$$= E(p', w, u') + \int_{p'}^{p^0} E_p(p, w, u')dp$$
$$- E(p^0, w, u^0)$$
$$= \Pi(p^l, w, \tau') + \int_{p^l}^{p'} \Pi_p(p, w, \tau')dp$$
$$+ \int_{p'}^{p^0} E_p(p, w, u')dp - E(p^0, w, u^0)$$
$$= x^r + p^l y^r + \int_{p^l}^{p'} \Pi_p(p, w, \tau')dp$$
$$+ \int_{p'}^{p^0} E_p(p, w, u')dp - (x^0 + p^0 y^0)$$
$$= \int_{p^l}^{p'} \Pi_p(p, w, \tau')dp$$
$$+ \int_{p'}^{p^0} E_p(p, w, u')dp$$
$$+ (x^r + p^h y^r) - (x^0 + p^0 y^0)$$
$$- (p^h - p^l)y^r$$
$$= \int_{p^l}^{p'} \Pi_p(p, w, \tau')dp$$
$$+ \int_{p'}^{p^0} E_p(p, w, u')dp$$
$$+ \int_{p^0}^{p^h} E_p(p, w, u^0)dp - (p^h - p^l)y^r.$$

Here, p^l is the price at point g and p^h is that at point c, so that the first integral of the final expression in equation (A1) is the area to the left of MRT′ between p^l and p', the second integral is the area to the left of MRS′ between p' and p^0, the third area is the integral to the left of MRS^0 between p^0 and p^h, and the final term subtracts the area to the left of line cg. Thus, EV is the shaded area $caebg$, and CV can be similarly established to equal area $cadbg$.

Now consider the equivalence of technical change $(x_8 - x_4)$ and area $gcaf$:

$$\text{(A2)}\quad \text{TC} \equiv \Pi(p^0, w, \tau') - \Pi(p^0, w, \tau^0)$$
$$= \left[p^l \Pi_p(p^l, w, \tau') + \int_{p^l}^{p^0} \Pi_p(p, w, \tau')dp\right]$$
$$- \left[p^h E_p(p^h, w, iu^0) - \int_{p^0}^{p^h} E_p(p, w, u^0)dp\right]$$
$$= p^l y^r + \int_{p^l}^{p^0} \Pi_p(p, w, \tau')dp$$
$$- p^h y^r + \int_{p^0}^{p^h} E_p(p, w, u^0)dp$$
$$= \int_{p^l}^{p^0} \Pi_p(p, w, \tau')dp$$
$$+ \int_{p^0}^{p^h} E_p(p, w, iu^0)dp - (p^h - p^l)y^r.$$

The first integral is the area to the left of the MRT′ curve from point g to point f, the second integral is the area to the left of the MRS^0 curve from point a to point c, and the final term subtracts the area to the left of line segment cg, leaving the area $gcaf$ as the area equivalent to the technical change.

Finally, consider the equivalence of the factor productivity change (FPC) and the area $gcab$:

$$\text{(A3)}\quad \text{FPC} \equiv \Pi(p^0, w, \tau^0)\tfrac{1}{2}[(k^0 + \hat{k}')d\ln y$$
$$+ (k_0^0 + \hat{k}_0')d\ln x_0]$$
$$= \tfrac{1}{2}[p^0 y' - p^0 y^0 + p' y'$$
$$- p' y^0 + 2(x_0' - x_0^0)]$$
$$= \tfrac{1}{2}[[x_0'] + p^0 y' - \Pi(p^0, w, \tau^0)$$
$$- (x_0^0 + p' y^0) + \Pi(p', w, \tau')]$$
$$= \tfrac{1}{2}[[\Pi(p', w, \tau'] - p' y'] + p^0 y'$$
$$+ (p^h - p^l)y^r - \Pi(p^0, w, \tau^0)$$
$$- (p^h - p^l)y^r + \Pi(p', w, \tau')$$
$$- [\Pi(p^0, w, \tau^0) - p^0 y^0]$$
$$- p' y^0 + (p^h - p^l)y^r - (p^h - p^l)y^r]$$
$$= \tfrac{1}{2}[\Pi(p', w, \tau') - \Pi(p^l, w, \tau')$$
$$+ (p^0 - p')y' + E(p^h, w, iu^0)$$
$$- E(p^0, w, u^0) - (p^h - p^l)y^r$$
$$\times \Pi(p', w, \tau') - \Pi(p^l, w, \tau')$$
$$+ (p^0 - p')y^0 + E(p^h, w, u^0)$$
$$- E(p^0, w, u^0) - (p^h - p^l)y^r]$$
$$= \int_{p^l}^{p'} \Pi_p(p, w, \tau')dp$$
$$+ \tfrac{1}{2}(p^0 - p')(y^0 + y')$$
$$+ \int_{p^0}^{p^h} E_p(p, w, u^0)dp - (p^h - p^l)y^r$$

Here, the first integral is the area to the left of MRT′ between point g and point b, the second term is the area to the left of line segment ab, the third term is the area to the left of MRS^0 between points a and c, and the final term subtracts the area to the left of line segment cg, leaving area $gcab$ as the area corresponding to the factor productivity change.